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20. Proposed by J. K. ELLWOOD, A. M., Principal of Colfax School, Pittsburg, Pennsylvania.

At what price must the government sell 5% $100 bonds to run 10 years, interest payable annually (quarterly), to make them equivalent to 3% bonds at par to run 10 years, interest payable annually (quarterly)? At what price if interest be paid semi-annually?

Solution by THEODORE L. DeLAND, Examiner, United States Civil Service Commission, Washington, D. C.

Let the symbols be as follows: $x=$ price paid for $1 of the 5% bond; $r=3\%$, the rate the investor realizes; $t=$ the time the bond runs in years; $n=$ the number of times the interest is paid in 1 year; $r_1=$ the rate the bond draws per annum.

Following the United States Rule for Partial Payments and we have, *after each interest payment*, *amounts due* as follows:

First:— $x\left(1+\frac{r}{n}\right)-\frac{r_1}{n}$.

Second:— $x\left(1+\frac{r}{n}\right)^2-\frac{r_1}{n}\left(1+\frac{r}{n}\right)-\frac{r_1}{n}$.

Third:— $x\left(1+\frac{r}{n}\right)^3-\frac{r_1}{n}\left(1+\frac{r}{n}\right)^2-\frac{r_1}{n}\left(1+\frac{r}{n}\right)-\frac{r_1}{n}$.

We can now determine the law of the series by inspection; and we also know that after the last interest payment is made there is due and payable the principal of the bond or $1; and we also know that after the interest and principal are paid there is nothing due. We form the general equation as follows:

$$x\left(1+\frac{r}{n}\right)^{nt}-\frac{r_1}{n}\left(1+\frac{r}{n}\right)^{nt-1}-\frac{r_1}{n}\left(1+\frac{r}{n}\right)^{nt-2}-\ldots\ldots-\frac{r_1}{n}\left(1+\frac{r}{n}\right)-\frac{r_1}{n}-1=0.$$

$$\therefore x\left(1+\frac{r}{n}\right)^{nt}=\frac{r_1}{n}\left[\left(1+\frac{r}{n}\right)^{nt-1}+\left(1+\frac{r}{n}\right)^{nt-2}+\ldots\ldots+\left(1+\frac{r}{n}\right)+1\right]+1.$$

The function within the brackets is a finite, determinable, equirational descending progression. Reverse the order of the terms of the function to make the series an ascending series, and to escape negative exponents in the result, and we have:

$$x\left(1+\frac{r}{n}\right)^{nt}=\frac{r_1}{n}\left[1+\left(1+\frac{r}{n}\right)+\left(1+\frac{r}{n}\right)^2+\ldots\ldots\right.$$
$$\left.+\left(1+\frac{r}{n}\right)^{nt-2}+\left(1+\frac{r}{n}\right)^{nt-1}\right]+1.$$

We observe in the series that the ratio is $\left(1+\frac{r}{n}\right)$, and that the last term is, $\left(1+\frac{r}{n}\right)^{nt-1}$. Sum it and we have:

$$x\left(1+\frac{r}{n}\right)^{nt}=\frac{r_1}{n}\left[\frac{\left(1+\frac{r}{n}\right)\left(1+\frac{r}{n}\right)^{nt-1}-1}{\left(1+\frac{r}{n}\right)-1}\right]+1.$$

$$\therefore x\left(1+\frac{r}{n}\right)^{nt}=\frac{r_1}{r}\left[\left(1+\frac{r}{n}\right)^{nt}-1\right]+1.$$

$$\therefore x\left(1+\frac{r}{n}\right)^{nt}=\frac{r_1\left(1+\frac{r}{n}\right)^{nt}}{r}-\frac{r_1}{r}+1.$$

$$\therefore x\left(1+\frac{r}{n}\right)^{nt}=\frac{1}{r}\left[r_1\left(1+\frac{r}{n}\right)^{nt}-(r_1-r)\right].$$

$$\therefore x=\frac{1}{r}\left[r_1-\frac{r_1-r}{\left(1+\frac{r}{n}\right)^{nt}}\right]\text{................(A).}$$

The above is the general equation in its simplest form. Substitute values from the problem, observing that $n=1$, and we have:

$$x=\frac{1}{.03}\left[.05-\frac{.05-.03}{(1.03)^{10}}\right]\text{........(B).}$$

Releive the equation of decimals and we have:

$$x=\frac{1}{3}\left[5-\frac{2}{(1.03)^{10}}\right]\text{................(C).}$$

The numerical computations are as follows:

Log 2=..................................0.3010300.
10 log 1.03=0.01283722×10...................0.1283722.
Log^{-1} 1.488188=...........................0.1726578.

5.000000−1.488188=3.511812. From (C), $x=\frac{1}{3}(5-1.488188)=\frac{1}{3}$ of 3.511812 =1.170604.

$\therefore x=\$1.170604$; or $100x=\$117.06$, the price of a $100-bond.

The general equation (A) can also be applied to the solution of the semi-annual bond. If we apply it to a quarterly bond we solve the government problem which confronted the Secretary of the Treasury when he placed the late $50,000,000-loan on the market; and which has given rise to so much hostile criticism from newspapers and actuaries. The figures given by Secretary Carlisle that the price, $117.22\frac{3}{10}$, for a $100-bond at 5%, to realize 3% to the investor, is a just equivalent, may be shown as follows:

Substitute data for a quarterly bond from the problem, observing that $n=4$, and we have: $x=\frac{1}{3}\left[5-\frac{2}{(1.0075)^{40}}\right]$.

Log 2 ....0.3010300
40 log 1.0075=0.0032451×40= 0.1298040
Log^{-1} 1.48329=........................0.1712260.

$\therefore$ $x=\frac{1}{3}(5-1.48329)=\frac{1}{3}$ of $3.51671=1.17223$. $\therefore$ $100x=\$117.22\frac{3}{10}$, the price to be paid for a \$100-bond at 5%, interest payable quarterly for 10 years, to realize 3% per annum payable quarterly.

Also solved by *J. H. DRUMMOND, H. C. WHITAKER*, and *G. B. M. ZERR*.

NOTE:—Problem 19 was also solved by *J. H. DRUMMOND, M. A. GRUBER, T. L. DeLAND, J. F. W. SHEFFER, G. B. M. ZERR*, and *F. P. MATZ*.

PROBLEMS.

26. Proposed by ALVIN E. SCHMIDT, Winesberg, Ohio.

Show that $abc>(a+b-c)(a+c-b)(b+c-a)$ unless $a=b=c$.

27. Proposed by A. H. BELL, Hillsboro, Illinois. (The problem from H. C. WILKS, Skull Run, Virginia.

An oarsman in rowing a boat down stream 7 miles from A to B and then back requires 12 minutes longer time, than commencing from B, and rowing up and back; the rate of speed for the 1st half of the time is 5 miles per hour, and for the 2nd half of the time is $4\frac{1}{2}$ miles per hour. Required the current.

28. Proposed by H. W. DRAUGHON, Clinton, Louisiana.

The working capacity of a horse is constant between the ages of a and b years, and decreases at a constantly accelerated rate from the age of b years to that of c years, becoming 0 at the latter age. If the value of the horse at the age of a years is d, give a formula for finding his value at any subsequent time.

Solutions to these problems should be received on or before September 1st.

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GEOMETRY.

Conducted by B.F.FINKEL, Kidder, Missouri. All contributions to this department should be sent to him.

SOLUTIONS TO PROBLEMS.

14. Proposed by HENRY HEATON, M. S., Atlantic, Iowa.

Through two given points to pass four circles tangent to two given circles.

Solution by the PROPOSER.

In the figure, C and C' the intersection of the common tangents to the two circles are known as the external and internal centers of similitude.

It is not necessary to demonstrate here the following well known properties: